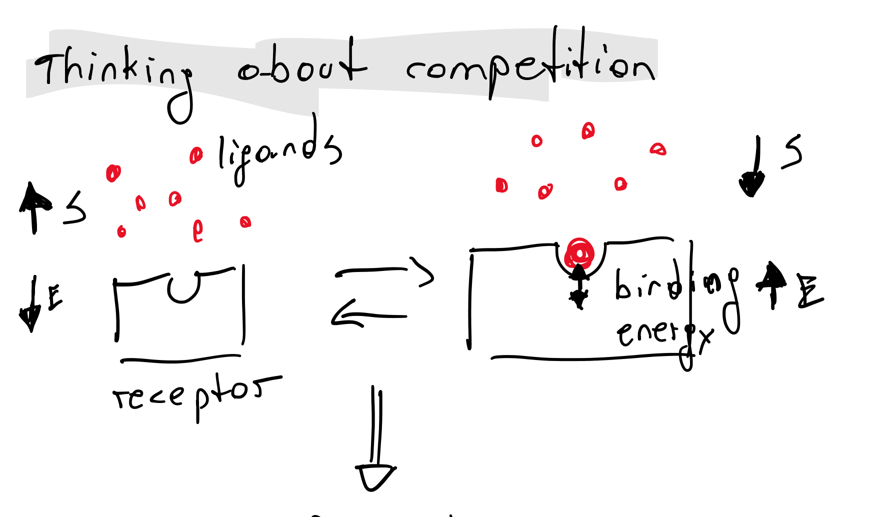
In-class Lecture 4 - Scaling, Part 2

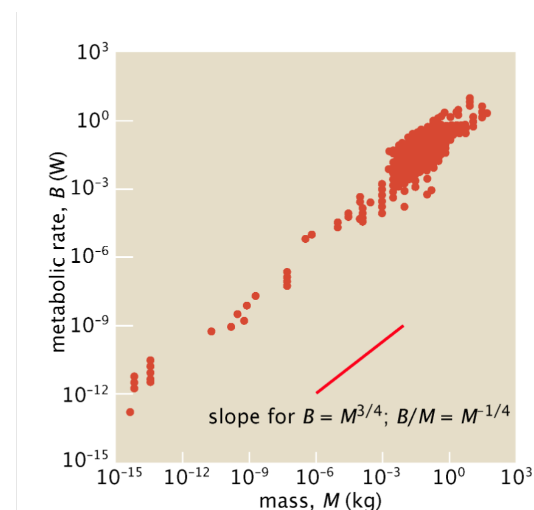
Tuesday, January 25, 2022 12:43 PM

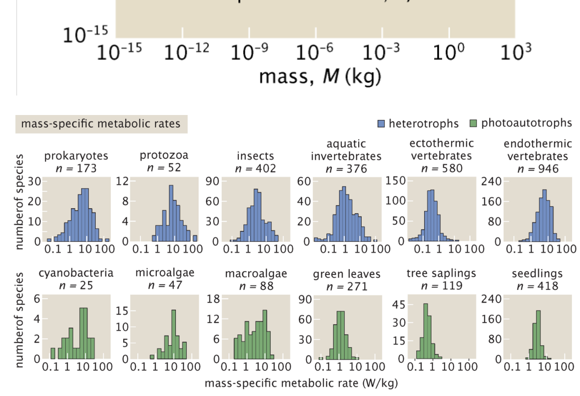
Thinking about competition



prob. of binding is determined by the competition between binding and entropy

Mass-specific metabolic rates





Estimate E. coli metabolic rate

Note to class: I decided to consider only the energy to assemble proteins

$$\frac{\text{energy consumed by E. coli to make another cell}}{\text{cell cycle time}} = \frac{\text{energy associated w/ putting together the proteins}}{\text{cell cycle time}}$$

We know that I pay 4 ATPs/a.a. assembled:

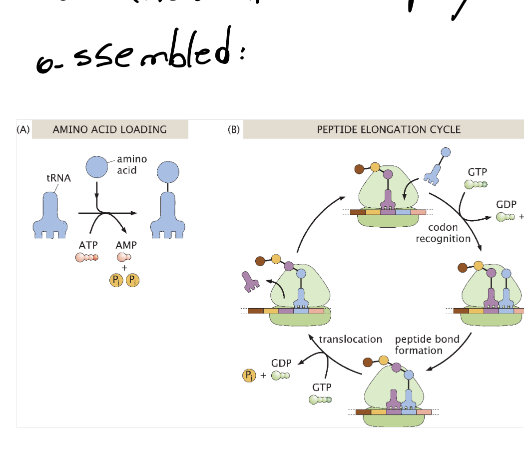


Table 3. ATP requirement for the formation of microbial cells from preformed monomers (glucose, amino acids and nucleic acid bases). Formation of lipids from acetate or glucose are separately considered.

Macromolecule	Amount of monomer moles × 10^{-4}/g cells	ATP required per monomer moles/mole: Lipids formed from Acetate	ATP required per monomer moles/mole: Lipids formed from Glucose	ATP required moles × 10^{-4}/g cells: Lipids formed from Acetate	ATP required moles × 10^{-4}/g cells: Lipids formed from Glucose
Polysaccharide	10.26	2	2	20.52	20.52
Protein	47.85	4	4	191.40	191.40
Lipid	1.40	33	1	46.20	1.40
RNA[1]	4.60	5	5	23.00	23.00
DNA	0.96	6	6	5.76	5.76
Turnover RNA				13.90	13.90
Total				300.78	255.98
ATP required for transport of					
Amino acids				47.85	47.85
Acetate				22.40	–
Potassium ions				1.92	1.92
Phosphate				7.74	7.74
Total ATP requirement				380.69	313.49
$Y_{ATP}^{MAX} = \frac{10\,000}{\text{Total ATP requirement}}$				26.4	31.9

[1] For the calculation of the RNA content the UMP content (Table 1) has been taken as basis.

energy to pay the workers $= 3\cdot 10^6$ prots $\cdot\, 300 \frac{a.a.}{prot} \cdot 4 \frac{ATP}{a.a.}$

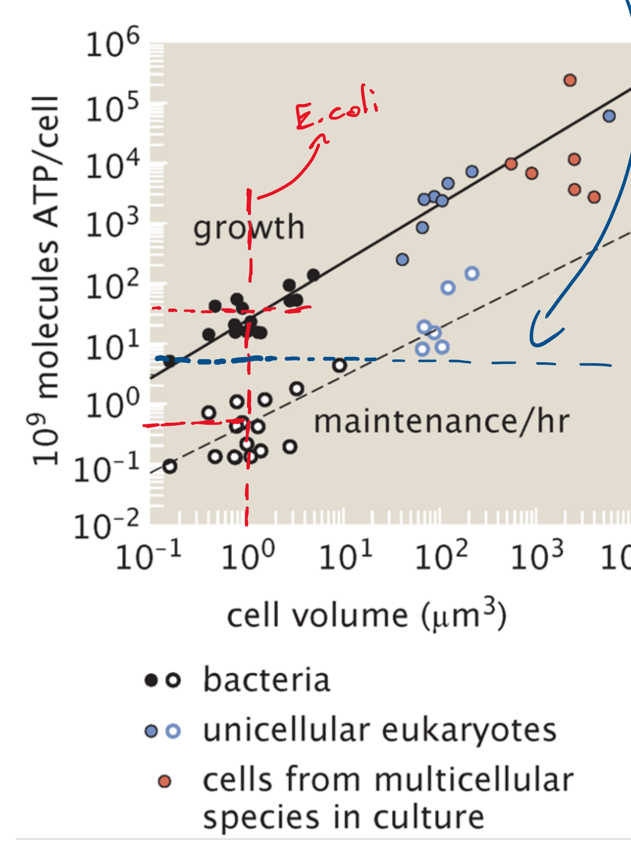
$= 9\cdot 10^8 \cdot 4$ ATP

$\approx 4\cdot 10^9$ ATP

Volume-specific energy required

$$M_V = \frac{4\cdot 10^9 \text{ ATP}}{1\,\mu m^3}$$

we are in the right range!



We see that there's a good agreement between our estimated metabolic rate for growth and that from Lynch + Marinov.

To calculate a rate, divide by the cell cycle

$$\text{Metabolic rate} = \frac{4\cdot 10^9 \text{ ATP}}{\mu m^3} \Big/ \underbrace{1000\,s}_{\sim 20\,min}$$

$$= 4\cdot 10^6 \frac{ATP}{\mu m^3\, s}$$

How do you make all this ATP?

⇒ ATP synthase:

spins @ $100 \frac{\text{revolutions}}{s}$

$= 100$ Hz

$= 6000 \frac{\text{revol}}{\text{min}}$

$= 6000$ RPM

comparable to jet engine!

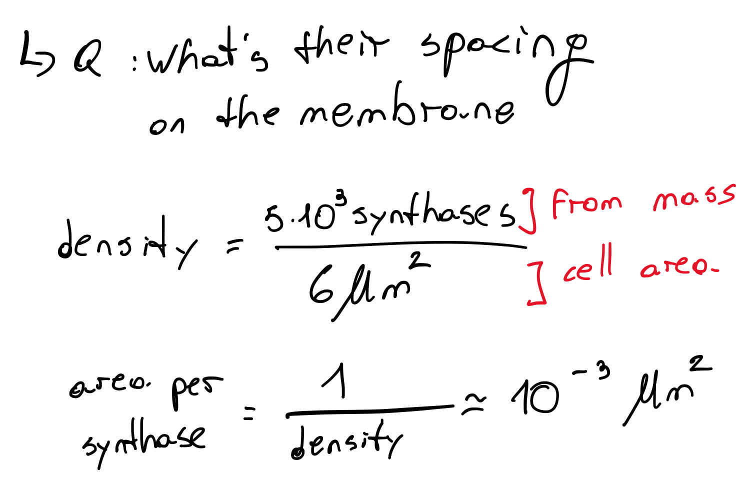
makes 3 ATP/revolution

How many ATP synthases do I need to support this maintenance metabolic rate?

$$\underbrace{4\cdot 10^6 \frac{ATP}{s}}_{\text{total ATP for } 1\mu m^3 \text{ E. coli}} \Big/ \underbrace{100 \frac{ATP}{s}}_{\text{rate per synthase}} \simeq 4\cdot 10^4 \text{ synthases}$$

not too bad!

Mass spec data:



↳ Q: what's their spacing on the membrane

$$\text{density} = \frac{5\cdot 10^3 \text{ synthases}}{6\,\mu m^2}$$] from mass spec] cell area

$$\text{area per synthase} = \frac{1}{\text{density}} \approx 10^{-3}\,\mu m^2$$

$$\text{synthase spacing} \approx \sqrt{\text{area per synthase}} \approx \frac{1}{\sqrt{100}\sqrt{10}}\,\mu m$$

$$\approx \frac{1}{10\cdot 3}\,\mu m \approx \underbrace{30\,nm}$$

spacing is comparable to size! They're closely packed

We posit that there's a competition between cell volume and area in energy production and consumption

$$ATP_{needed} = 4\cdot 10^6 \frac{ATP}{\mu m^3 \cdot s} \cdot V_{cell}$$

$ATP_{produced}$ = calculate maximum production rate

$$\sigma_{synth} = \text{surface density of synthases} = \frac{1}{A_{synthase}}$$

↓

assume membrane closely packed with ATP synthase molecules

$$ATP_{produced} = \sigma_{synth} \cdot 100 \frac{ATP}{s} \cdot A_{cell}$$

$$= \frac{1}{\pi\, 25\,nm^2} \cdot 100 \frac{ATP}{s} A_{cell}$$

$$= \frac{1}{\cancel{\pi}\,\cancel{25}\cdot 10^{-6}\,\mu m^2} \cdot \cancel{100}^{\,4} \frac{ATP}{s} A_{cell}$$

$$= 10^6 \frac{ATP}{s\,\mu m^2}$$

$$ATP_{needed} = 4\cdot 10^6 \frac{ATP}{\mu m^3 \cdot s} \cdot \frac{4}{3}\pi \cdot R^3$$

$$ATP_{produced} = 10^6 \frac{ATP}{\mu m^2 s} \cdot 4\pi R^2$$

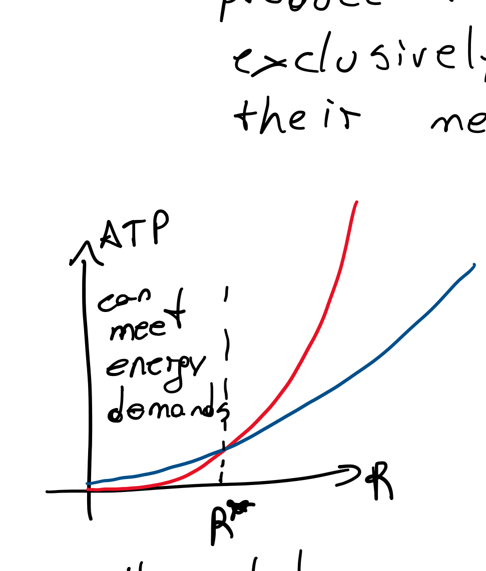
Let's find the R for which consumption and production become equal

$$\cancel{4}\cdot\cancel{10^6} \frac{\cancel{ATP}}{\mu m^{\cancel{3}}\,\cancel{s}} \frac{\cancel{4}}{\cancel{3}} \cancel{\pi} R^{\cancel{3}} = \cancel{10^6} \frac{\cancel{ATP}}{\cancel{\mu m^2}\,\cancel{s}} \cancel{4\pi R^2}$$

$$R \simeq 1\,\mu m$$

⇓

cells much larger than $1\,\mu m$ can't produce their ATP exclusively on their membrane!



↳ Make this plot