In class Lecture 22 - Dynamical Systems and the Lambda Switch, Part II

Thursday, April 7, 2022 10:28 AM

We start w/ our non-dimensional eqns. for mutual repression

$$\frac{d\tilde{r}_1}{d\tilde{t}} = \frac{\tilde{r}}{1+2\tilde{r}_2+\tilde{r}_2^2\omega} - \tilde{r}_1$$

$$\frac{d\tilde{r}_2}{d\tilde{t}} = \frac{\tilde{r}}{1+2\tilde{r}_1+\tilde{r}_1^2\omega} - \tilde{r}_2$$

two params

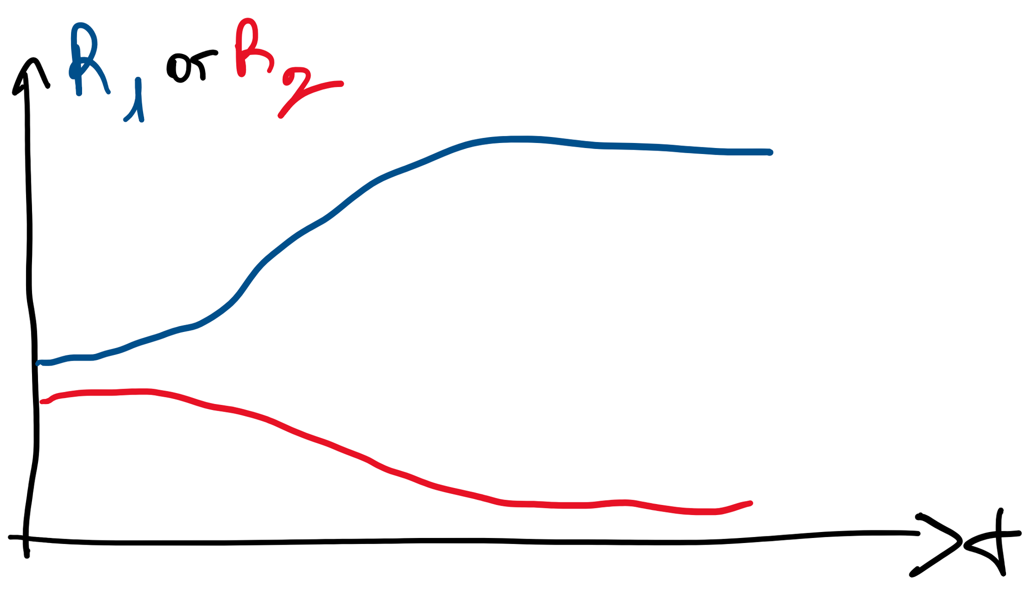
with

$\tilde{t} = t\cdot\gamma$

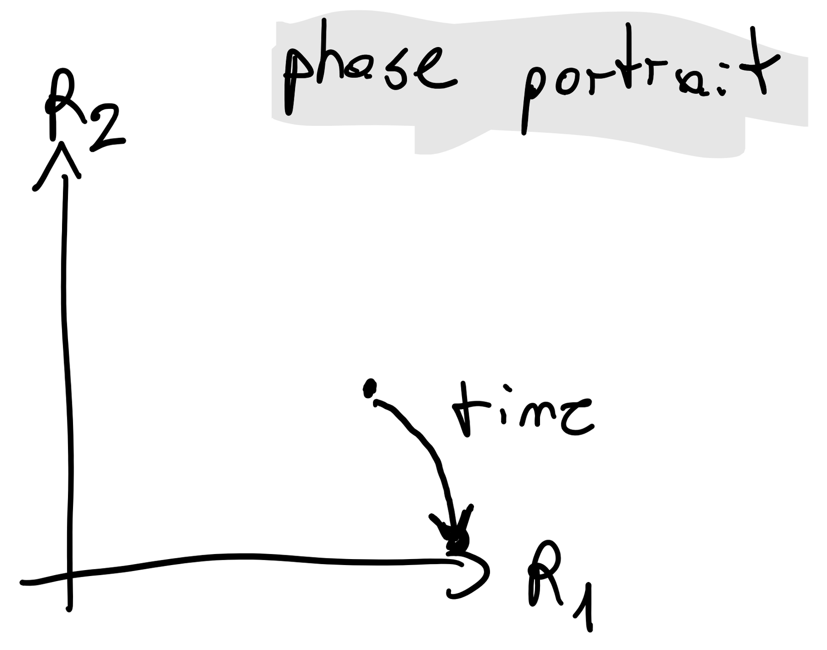
$\tilde{r}_1 = \frac{R_1}{K_d}$ $\tilde{r}_2 = \frac{R_2}{K_d}$

$\tilde{r} = \frac{r}{K_d\gamma}$

What can we learn about the system's dynamics w/o doing much math/simulations?



phase portrait



Calculate the nullclines

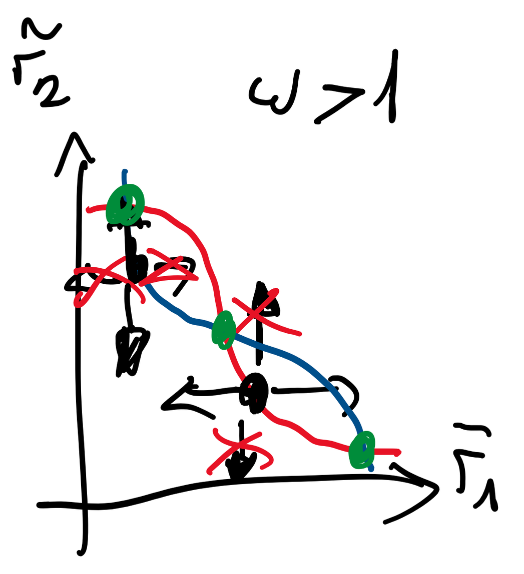
nullclines

$$\frac{d\tilde{r}_1}{d\tilde{t}} = 0 = \frac{\tilde{r}}{1+2\tilde{r}_2+\tilde{r}_2^2\omega} - \tilde{r}_1$$

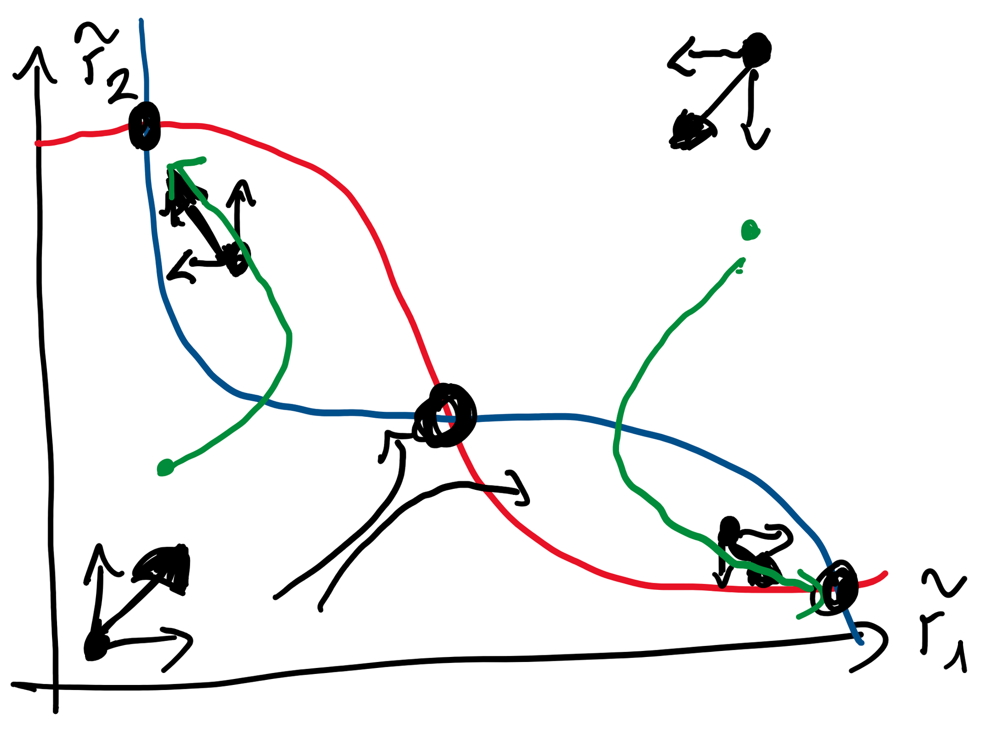
$$\frac{d\tilde{r}_2}{d\tilde{t}} = 0 = \frac{\tilde{r}}{1+2\tilde{r}_1+\tilde{r}_1^2\omega} - \tilde{r}_2$$

$$\Downarrow$$

$$\tilde{r}_2 = \frac{\tilde{r}}{1+2\tilde{r}_1+\tilde{r}_1^2\omega}$$



Phase portrait



— $\frac{d\tilde{r}_1}{d\tilde{t}} = 0$

— $\frac{d\tilde{r}_2}{d\tilde{t}} = 0$

$$\frac{d\tilde{r}_1}{d\tilde{t}} = \frac{\tilde{r}}{1+2\tilde{r}_2+\tilde{r}_2^2\omega} - \tilde{r}_1$$

$$\frac{d\tilde{r}_2}{d\tilde{t}} = \frac{\tilde{r}}{1+2\tilde{r}_1+\tilde{r}_1^2\omega} - \tilde{r}_2$$

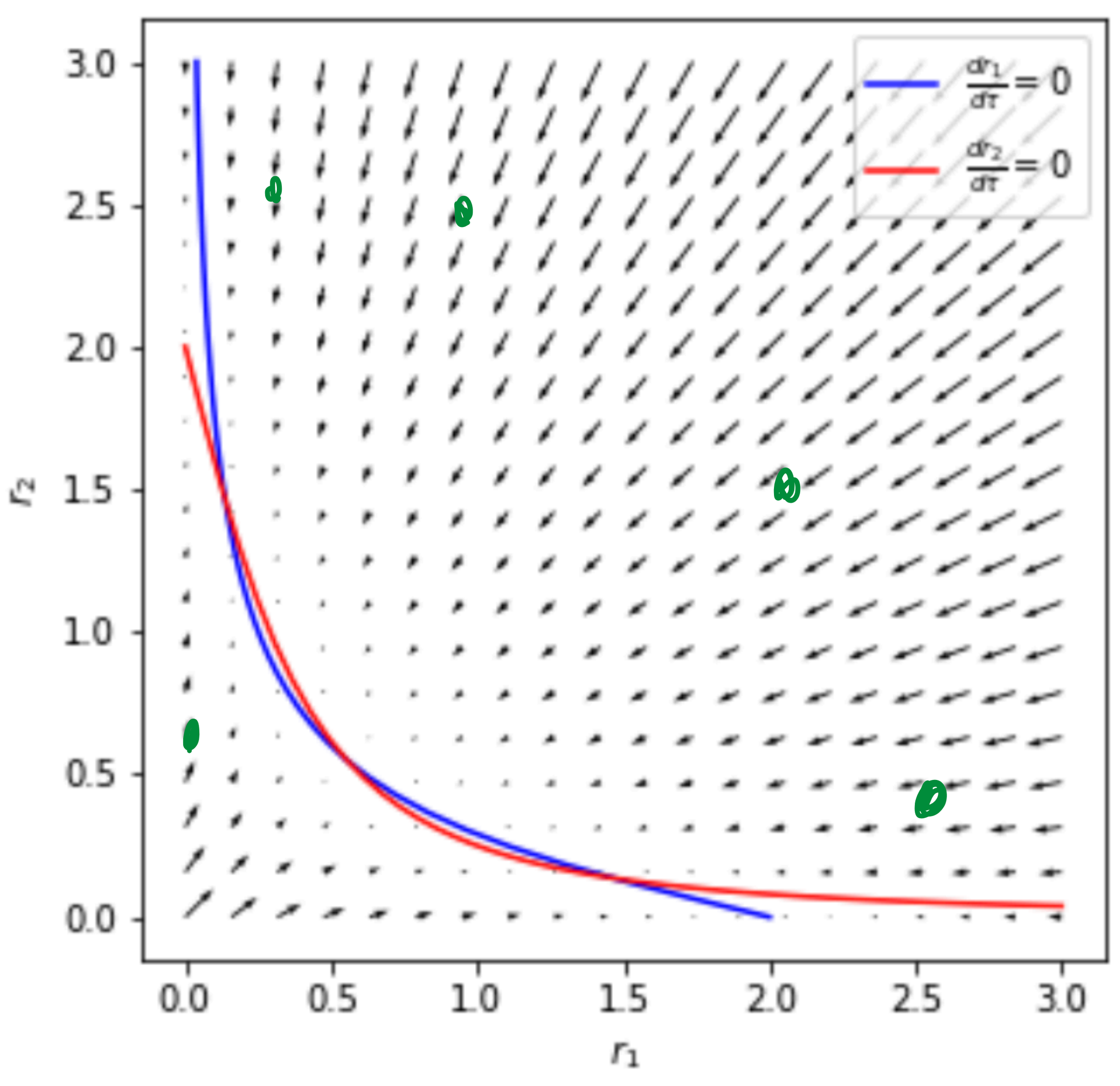
if r_1, r_2 are large

$\frac{d\tilde{r}_1}{d\tilde{t}} < 0$ $\frac{d\tilde{r}_2}{d\tilde{t}} < 0$

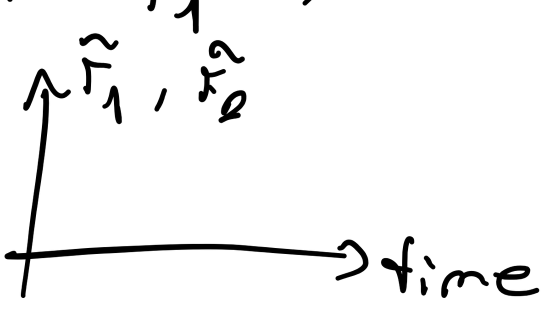
if $r_1, r_2 \simeq 0$

$\frac{d\tilde{r}_1}{d\tilde{t}} > 0$ $\frac{d\tilde{r}_2}{d\tilde{t}} > 0$

We computed the vector field in Python



- Calculate $\tilde{r}_1(t)$ and $\tilde{r}_2(t)$ given some init cond and given $\omega = 5$, $\tilde{r} = 2$
- Plot $\tilde{r}_1(t)$ and $\tilde{r}_2(t)$



- Plot $(\tilde{r}_1, \tilde{r}_2)$ on the phase portrait

$$\tilde{r}_1(\tilde{t}+\Delta\tilde{t}) = \tilde{r}_1(\tilde{t}) + \frac{\tilde{r}}{1+2\tilde{r}_2(\tilde{t})+\tilde{r}_2^2(\tilde{t})\omega}\cdot\Delta\tilde{t} - \gamma\,\tilde{r}_1(\tilde{t})\Delta\tilde{t}$$

$$\tilde{r}_2(\tilde{t}+\Delta\tilde{t}) = \tilde{r}_2(\tilde{t}) + \frac{\tilde{r}}{1+2\tilde{r}_1(\tilde{t})+\tilde{r}_1^2(\tilde{t})\omega}\cdot\Delta\tilde{t} - \gamma\,\tilde{r}_2(\tilde{t})\Delta\tilde{t}$$