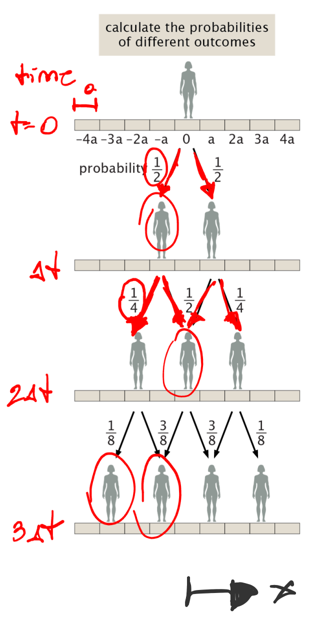
In class - Diffusion by coin flips

Thursday, February 22, 2024 1:55 PM



$\langle x \rangle$

$\emptyset$

$\frac{1}{2}(-a)+\frac{1}{2}(+a)=\emptyset$

$\frac{1}{4}(-2a)+\frac{1}{2}(0)+\frac{1}{4}2a=\emptyset$

$\emptyset$

$\langle x^2 \rangle$

$\emptyset$

$\frac{1}{2}(-a)^2+\frac{1}{2}(a)^2=a^2$

$\frac{1}{4}(-2a)^2+\frac{1}{2}(0)^2+$
$+\frac{1}{4}(2a)^2=\frac{1}{4}\cdot 4a^2+\frac{1}{4}\cdot 4a^2$

$=2a^2$

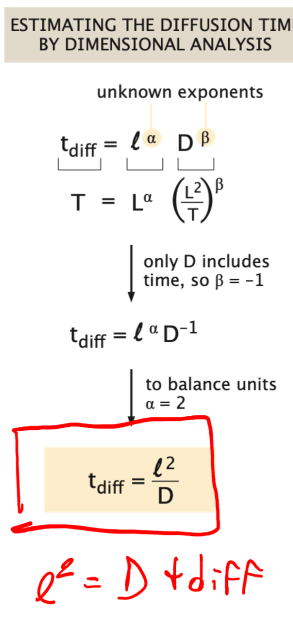
$=3a^2$

$\sqrt{\langle x^2 \rangle}$ = RMS
root mean square

We conclude:

$$\langle x^2 \rangle = N a^2 = \frac{T}{\Delta t} a^2 = \frac{a^2}{\Delta t}\cdot T$$

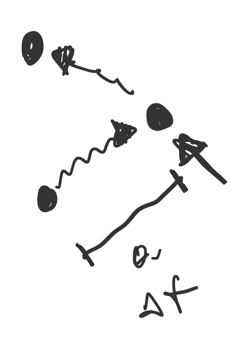
N → # of coin flips $= \frac{T}{\Delta t}$ (T → total time, Δt → time step)

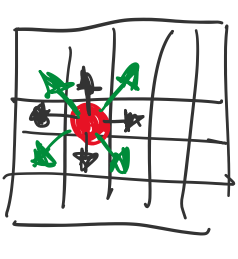


$\ell^2 = D\, t_{diff}$

$\langle x^2 \rangle = D \cdot T = \frac{a^2}{\Delta t}\cdot T$

$\Rightarrow D = \frac{a^2}{\Delta t}$ (a^2 → size of jump, Δt → time between jumps)





Next: simulate the trajectory of a random walker

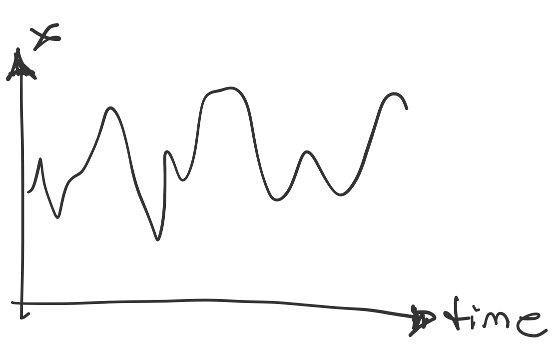
$x = [x_0, x_1, \ldots, x_{N-1}]$ = position of random walker

N → # of coin flips

At every time step → right: $N[i] = N[i-1]+1$ or → left

0 0.5 1 (-1 | $+1$)

$D_{Prot} = 10 \frac{\mu m^2}{s} = \frac{a^2}{\Delta t}$

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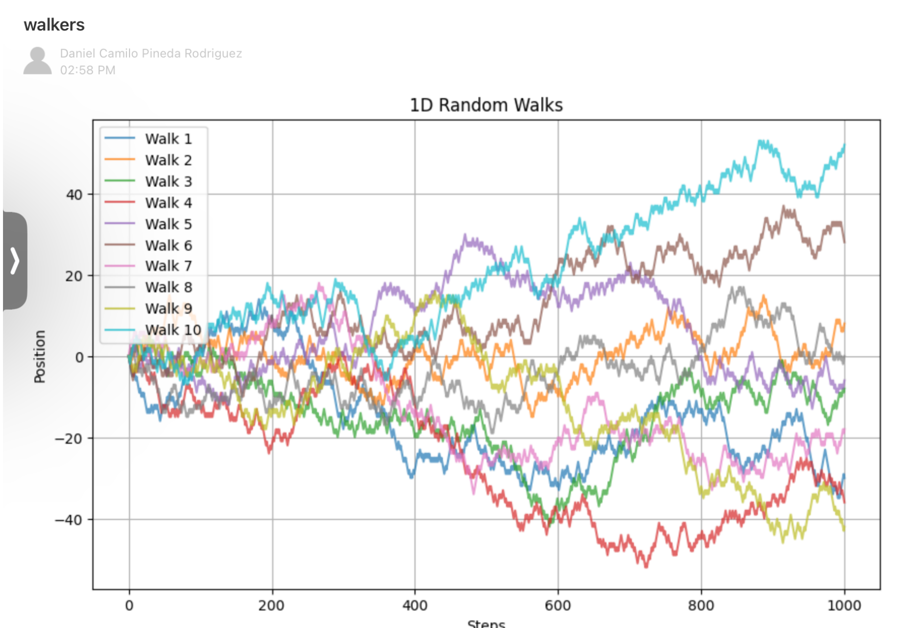
N[i=0] = 0
for i=1 to imax
  rand = random # (0 and 1)
  if rand > 0.5      (move to the right)
    N[i] = N[i-1] + 1
  else               (move to the left)
    N[i] = N[i-1] - 1
  end if
end for loop
```

Now do the simulation for 1000 walkers

walkers

Daniel Camilo Pineda Rodriguez 02:58 PM



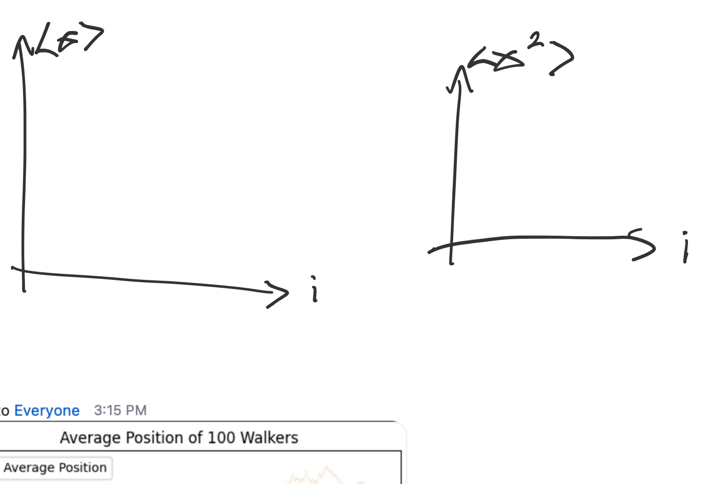
$\langle x \rangle$ $\langle x^2 \rangle$

$N_{walkers} \Rightarrow \langle x \rangle (i=2) = \sum_{j=1}^{N_{walker}} \langle x \rangle_j (i=2)\cdot \frac{1}{N_{walkers}}$ (j → walker j)

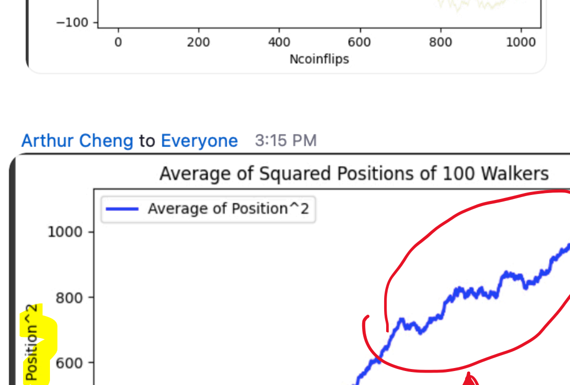
$x = [x_0, x_1, \ldots, x_{N-1}]$ (for 1 walker)

$$X = \begin{bmatrix} x_{00} & x_{01} & \cdots & x_{0N-1} \\ x_{10} & & \ddots & \vdots \\ \vdots & & & \\ x_{N_{walker}0} & \cdots & & x_{N_{walker}N-1} \end{bmatrix}$$

(first index → walker #, second index → time point #)



Arthur Cheng to Everyone 3:15 PM



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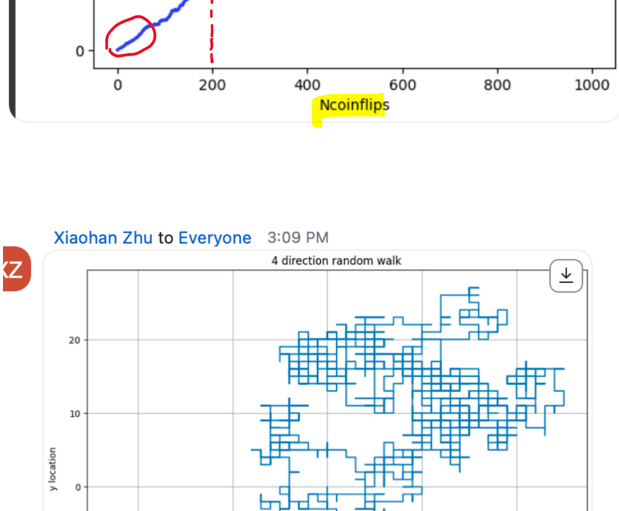
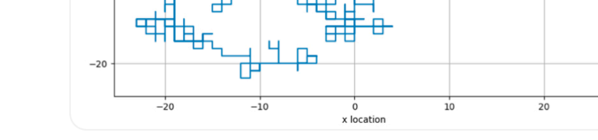


$\langle x^2 \rangle = a^2 N$ (a → 1)

$= N$

Xiaohan Zhu to Everyone 3:09 PM



0 0.25 0.5 0.75 1

$\langle x^2 + y^2 \rangle$