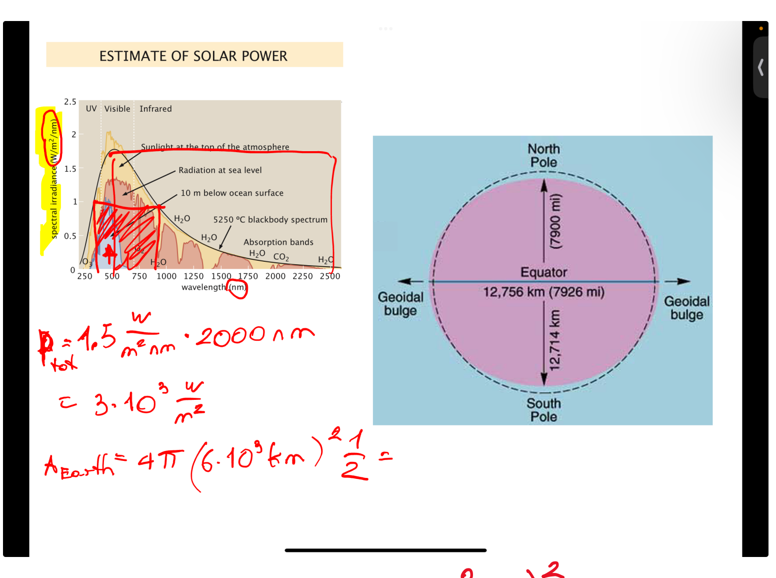
IN CLASS - Lecture 21: Defiance, Part II

Thursday, April 4, 2024 1:19 PM



$P_{tot} = 1.5 \frac{W}{m^2 nm} \cdot 2000\,nm$

$= 3 \cdot 10^3 \frac{W}{m^2}$

$A_{Earth} = 4\pi (6 \cdot 10^3 km)^2 \frac{1}{2} =$

$= 2\pi (6 \cdot 10^3 \cdot 10^3 m)^2$

$= 6 (6 \cdot 10^6 m)^2 = 6 \cdot 36 \cdot 10^{12} m^2 \simeq 200 \cdot 10^{12} m^2$

$= 2 \cdot 10^{14} m^2$

$P_{Earth} = P_{tot} A_{Earth} = 3 \cdot 10^3 \frac{W}{m^2} \cdot 2 \cdot 10^{14} m^2$

$= 6 \cdot 10^{17} W$

$E_{Earth} = P_{Earth} \cdot t_{day} = 6 \cdot 10^{17} \frac{J}{s} \, 24 \frac{h}{day} \cdot 3600 \frac{s}{h}$

$= 6 \cdot 10^{17} \cdot 2.4 \cdot 10 \cdot 3.6 \cdot 10^3 \, J/day$

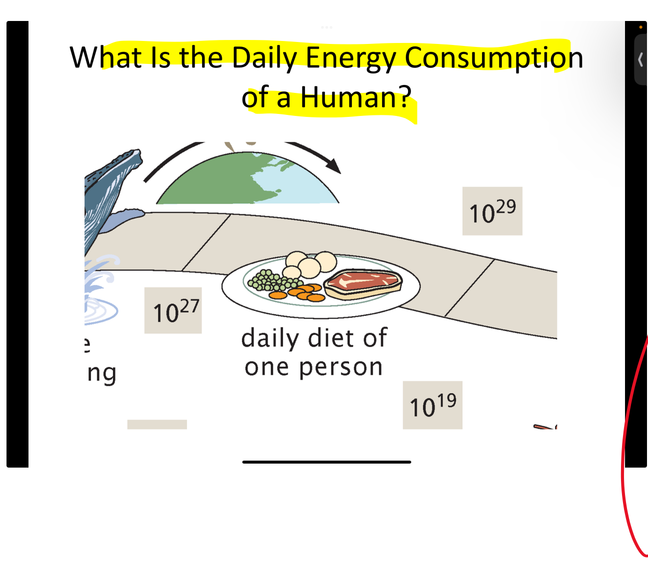
$= 10^{18} \cdot \text{few} \cdot 10 \cdot \text{few} \, 10^3 \frac{J}{day}$

$= 10^{23} \frac{J}{day} \cdot \frac{1 k_BT}{4\,pN nm} =$

$= 10^{23} \frac{Nm}{day} \cdot \frac{1 k_BT}{4 \cdot 10^{-12} N \cdot 10^{-9} m}$

$= \frac{10^{23} k_BT}{4 \cdot 10^{-21} \, day} = \frac{1}{4} \cdot 10^{44} \frac{k_BT}{day}$

$\simeq 2.5 \cdot 10^{43} \frac{k_BT}{day}$



$E = 2000\,kcal \cdot 4 \frac{J}{cal}$

$= 2 \cdot 10^6 cal \cdot 4 \frac{J}{cal}$

$= 8 \cdot 10^6 J \simeq 10^7 J$

$P = \frac{E}{1 day} = \frac{10^7 J}{10^5 s} =$

$= 10^2 W = 100W$

$E = 10^7 J \cdot \frac{1 k_BT}{4\,pN\,nm} = 10^7 N \cdot m \frac{1 k_BT}{4 \cdot 10^{-12} N \cdot 10^{-9} m}$

$= \frac{1}{4} \cdot 10^7 \cdot 10^{12} \cdot 10^9 k_BT = 0.25 \cdot 10^{28} k_BT$

$< 2.5 \cdot 10^{27} k_BT$

Thinking about the energy stored in an ion gradient

$S_{tot} = S_1 + S_2$

differential

$0 \leq dS_{tot} = d(S_1 + S_2) = dS_1 + dS_2 = 0$ ← in equil

2nd law of thermodynamics

$0 \leq dS_1 + dS_2 = \left(\frac{dS_1}{dN_1}\right) dN_1 + \left(\frac{dS_2}{dN_2}\right) dN_2$

$= \left(\frac{dS_1}{dN_1}\right) dN_1 - \left(\frac{dS_2}{dN_2}\right) dN_1$

$dN_1 = -dN_2$

chemical potential, μ

$0 \leq \left[\frac{dS_1}{dN_1} - \frac{dS_2}{dN_2}\right] dN_1$

$0 \leq (\mu_1 - \mu_2) dN_1$

this is the driving force for mass transport (or conversion)

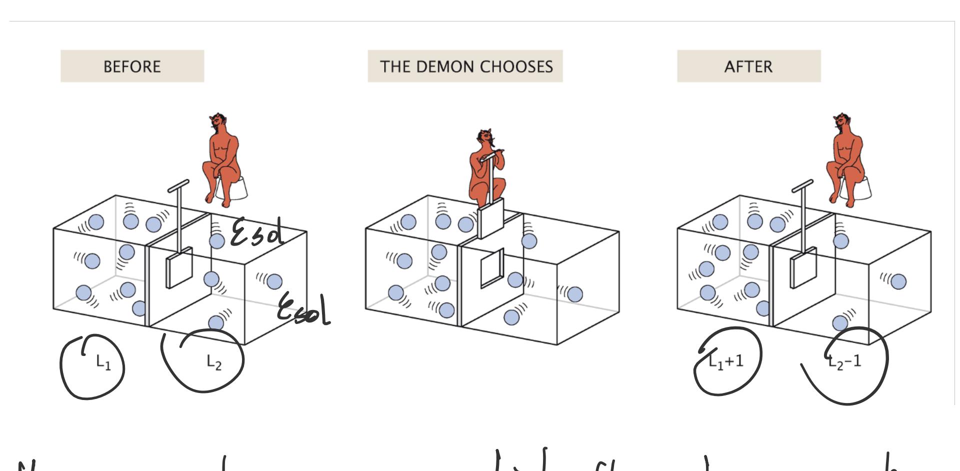
$\mu_1 > \mu_2 \Rightarrow dN_1 > 0$

$\mu_1 < \mu_2 \Rightarrow dN_1 < 0$

$\mu_1 = \mu_2 \Rightarrow dN_1 = 0$

equality of chemical potential = equilibrium or entropy maximization

Use these concepts for how the demon defies diffusion



How much energy did the demon have to pay to move one molec. from one box to the other?

$G_{initial}(L_1, L_2) =$ free energy $= U_{init.} - TS_{init.}$ ← $L_1 E_{sol} + L_2 E_{sol}$

$G_{final}(L_1+1, L_2-1) = U_{final} - TS_{final}$ ← $(L_1+1)E_{sol} + (L_2-1)E_{sol}$

assume they're equal

$\Delta G = G_{final} - G_{init} = -TS_{final} + TS_{init}$

$= -T[S(L_1+1, L_2-1) - S(L_1, L_2)]$

$k_B[\ln W(L_1+1) + \ln W(L_2-1)]$ $\quad k_B[\ln W(L_1) + \ln W(L_2)]$

$= -Tk_B\left[\ln \frac{\Omega^{L_1+1}}{(L_1+1)!} + \ln \frac{\Omega^{L_2-1}}{(L_2-1)!} - \ln \frac{\Omega^{L_1}}{L_1!} - \ln \frac{\Omega^{L_2}}{L_2!}\right]$

stadium seating

$= -Tk_B \ln\left[\frac{\Omega^{L_1+1} \, \Omega^{L_2-1}}{(L_1+1)! \, (L_2-1)!} \frac{L_1! \, L_2!}{\Omega^{L_1} \, \Omega^{L_2}}\right]$

$= -Tk_B \ln\left[\frac{\Omega}{(L_1+1)} \cdot \frac{L_2}{\Omega}\right] \simeq -Tk_B \ln \frac{L_2}{L_1}$

$L_1 + 1 \simeq L_1$

$\Delta G = -Tk_B \ln\left(\frac{L_2}{L_1}\right)$

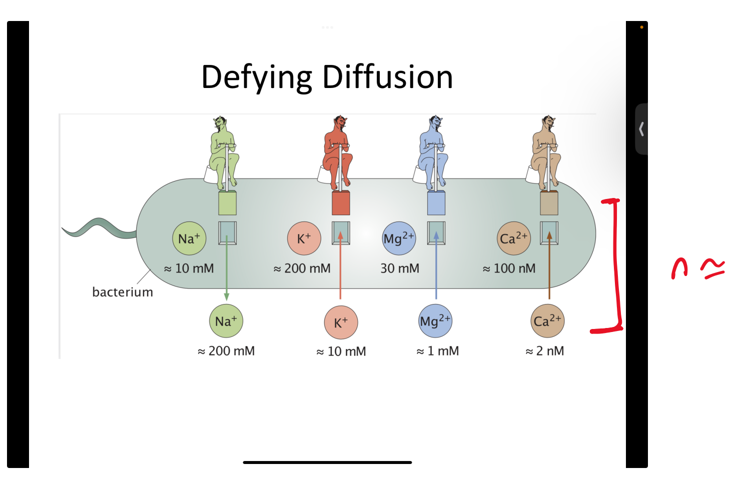
Now, let's assume: $L_2 = L_1 \cdot 10^n$

$\Delta G = -Tk_B \ln\left(\frac{L_1 \cdot 10^n}{L_1}\right) = -Tk_B \ln 10^n$

$= -Tk_B \cdot n \cdot \ln 10 = -Tk_B \cdot n \cdot 2.3$

$\ln 10 \sim 2.3$

2.3 k_BT paid for an order of magnitude diff. in concentration



$n \simeq 1$

$2.3\, k_BT = 0.1$ ATP/molec.

$\sim 20\, k_BT/ATP$