**Current Biology**

**Cell-Size Control and Homeostasis in Bacteria**

**Graphical Abstract**

Constant added mass and size convergence

**Authors**

Sattar Taheri-Araghi, Serena Bradde, ..., Massimo Vergassola, Suckjoon Jun

**Correspondence**

massimo@physics.ucsd.edu (M.V.), suckjoon.jun@gmail.com (S.J.)

**In Brief**

Taheri-Araghi et al. present extensive single-cell data from Gram-negative *E. coli* and Gram-positive *B. subtilis* showing that in both cases, cells add a constant volume, irrespective of birth size, and this automatically ensures size homeostasis.

**Highlights**

- Individual cells show systematic deviations from the population-level growth law
- Cells sense neither space nor time but add constant mass, irrespective of birth size
- The adder principle automatically ensures size homeostasis
- All measured distributions collapse when rescaled by their respective means
Cell-Size Control and Homeostasis in Bacteria

Sattar Taheri-Araghi, 1,7 Serena Bradde, 2,7 John T. Sauls, 1 Norbert S. Hill, 3 Petra Anne Levin, 4 Johan Paulsson, 5 Massimo Vergassola, 1,7 and Suckjoon Jun 1,6, 7
1Department of Physics, University of California San Diego, La Jolla, CA 92093, USA
2Initiative for the Theoretical Sciences, The Graduate Center, City University of New York, 365 Fifth Avenue, New York, NY 10016, USA
3Department of Molecular & Cell Biology, University of California, Berkeley, CA 94720, USA
4Department of Biology, Washington University, Saint Louis, MO 63130, USA
5Department of Systems Biology, Harvard Medical School, Longwood, MA 02115, USA
6Section of Molecular Biology, Division of Biological Science, University of California San Diego, La Jolla, CA 92093, USA

Summary

How cells control their size and maintain size homeostasis is a fundamental open question. Cell-size homeostasis has been discussed in the context of two major paradigms: “sizer,” in which the cell actively monitors its size and triggers the cell cycle once it reaches a critical size, and “timer,” in which the cell attempts to grow for a specific amount of time before division. These paradigms, in conjunction with the “growth law” [1] and the quantitative bacterial cell-cycle model [2], inspired numerous theoretical models [3–9] and experimental investigations, from growth [10, 11] to cell cycle and size control [12–15]. However, experimental evidence involved difficult-to-verify assumptions or population-averaged data, which allowed different interpretations [1–5, 16–20] or limited conclusions [4–9]. In particular, population-averaged data and correlations are inconclusive as the averaging masks causal effects at the cellular level. In this work, we extended a microfluidic “mother machine” [21] and monitored hundreds of thousands of Gram-negative Escherichia coli and Gram-positive Bacillus subtilis cells under a wide range of steady-state growth conditions. Our combined experimental results and quantitative analysis demonstrate that cells add a constant volume each generation, irrespective of their newborn sizes, conclusively supporting the so-called constant Δ model. This model was introduced for E. coli [6, 7] and recently revisited [9], but experimental evidence was limited to correlations. This “adder” principle quantitatively explains experimental data at both the population and single-cell levels, including the origin and the hierarchy of variability in the size-control mechanisms and how cells maintain size homeostasis.

Results

At the Population Level, New Experimental Data Confirm the Growth Law

Population-level parameters derived from our single-cell data followed established patterns for microbial growth known as the growth law [1]: the average newborn cell volume (νb) increased and the average generation time (τd) decreased, respectively, as the nutrient-imposed growth rate (λ) increased (newborn refers to the cells right after birth; Figure 1A). The newborn cell volume depended exponentially on the nutrient-imposed growth rate (hereafter referred to as growth rate, unless otherwise noted), (νb) = A exp[B(λ)], in quantitative agreement with the growth law [1] (Figure 1C, red symbols and line; A is the y intercept, and B is the slope of the red line). Moreover, newborn length (s0) and width (wb), averaged over the entire set of individual cells in each growth condition, also showed an exponential dependence on the average growth rate (λ) (Figure S1A available online).

The size of individual cells also increased exponentially as s(t) = s02(λt) (where x is the instantaneous elongation rate), and their width did not change significantly between birth and division (Figure S1B; [21]; hereafter, we use size and volume synonymously). The average instantaneous elongation rate was identical to the average growth rate of the population since (1/s ds/dt) = (x) ln2 = (1/τd) ln2 = (λ).

At the Single-Cell Level, Individual Cells Show Systematic Deviations from the Growth Law

Individual cells, however, exhibited intrinsic variability even under constant growth conditions, and we asked whether the quantitative relationship between the average size and the average growth rate also applied at the single-cell level. For example, the SDs of the growth rate and the newborn cell size were ~15% and ~14% of their respective means (Figure 1B). Therefore, when the growth-rate distributions for two different growth conditions partially overlapped as shown in Figure 1B, individual cells in the overlap region could have had the same growth rate λ = (ln 2)/τd. Thus, if the growth rate solely defined the cell’s growth physiology, individual cells with the same λ should have had on average the same size as described by the growth law (νb) = A exp[B(λ)]. We found this was not the case. For all seven growth conditions, the size versus growth rate measured from individual cells, νb versus λ, systematically deviated from the population-level growth law (Figure 1C, blue symbols and lines versus red symbols and line). This deviation indicates that, at the single-cell level, the size of individual cells is controlled by a mechanism that is different from the growth law (νb) = A exp[B(λ)] (see below).

Correlations of Growth and Size Parameters Contradict Both Sizer and Timer Models

The newborn cell size (s0) and the generation time (τd) of individual cells were negatively correlated (Figure 1D, left), which excluded the timer model of cell-size control. Otherwise, we would have seen constant τd with respect to s0. Furthermore, timer models showed instability when accounting for the
observed exponential growth of individual cells (Supplemental Information). The fact that cells born small take on average more time before they divide is in principle consistent with a sizer model. However, the strong positive correlations between the dividing size $s_d$ and $s_b$ (Figure 1D, right) ruled out the model because the sizer predicted that $s_d$ should be constant.

**Cells Instead Employ “Adder” Principle**

Our data instead support a model in which the size added between birth and division ($\Delta = s_d - s_b$) is constant for given growth conditions. We found that, although $\Delta$ varied significantly between growth conditions and also between individual cells, $\Delta$ was on average constant irrespective of the newborn size $s_b$ in each growth condition (Supplemental Information). In fact, the entire conditional distribution $P(\Delta | s_b)$ had the same shape as the nonconditional distribution $P(\Delta)$, and distributions of $\Delta$ from different experimental conditions collapsed onto a single curve when rescaled by their mean (Figure 2, right; Figure S2). The distribution of the size added in each generation, $\Delta$, was thus independent of the newborn cell size.

We also confirmed the constancy of $\Delta$ in two additional E. coli strains from our previous work (K12 MG1655 and B/r) [21] (Figure S3) and E. coli size mutants ($\Delta$pgm and ftsA*) [16]. Furthermore, we also confirmed the validity of the model in the Gram-positive B. subtilis (Figures 2B and 2C).

The collapse of the conditional distributions in Figure 2 established the constant $\Delta$ model, or adder (as opposed to

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**Figure 1. Growth Law at the Population Level and Systematic Deviations at the Single-Cell Level**

(A) Top: time series of a typical cell growing in a nutrient-rich medium. Bottom: sample images of dividing E. coli cells in steady-state exponential growth at 37°C in seven different growth media.

(B) Partially overlapping distributions of the growth rate and the newborn size measured from individual cells in two different growth conditions. The vertical lines show the population average values. Cells in the overlap region can have the same growth rate or newborn cell size.

(C) Population average of single-cell measurements demonstrates exponential dependence of newborn cell volume on the average growth rate (red). However, $s_b$ versus $\lambda$ of individual cells (binned data in empty blue circles; measured by following them from birth to division) shows systematic deviations from the average growth law. Thus, although the cells in the overlap region in (B) can have the same growth rate or newborn cell size, the size of individual cells are controlled by a mechanism that is different from the growth law. Otherwise, all blue symbols would have fallen on top of the red line.

(D) Correlations between rescaled growth parameters at the single-cell level with SDs from the entire set of E. coli data. Left: generation time versus size at birth. Middle: elongation rate versus size at birth. Right: size at division versus size at birth. Dashed lines indicate predictions from the adder principle from this work. The first correlation falsifies the timer model, whereas the last correlation falsifies the sizer model.

See also Figure S1.
“timer” or “sizer”). Next, we explain quantitatively consequences of this adder principle on cell-size homeostasis.

Adder Ensures Size Homeostasis
An immediate consequence of addition of constant $D$ is that it automatically ensured size homeostasis because at every cell division, the cell approached (albeit passively) the population average as illustrated in Figure 3 A (data depict the average behavior in all growth conditions). If a cell born at size $s_b = S_0 + \delta S_0$ stochastically added an uncorrelated size $D$ and divided in the middle with some precision, then the daughter sizes on average were $S_0 + \delta S_0/2$. After $n$ consecutive divisions, the original size deviation of the newborn cell on average decreased as $\delta S_0/2^n$ (Figure 3A). The size homeostasis principle was confirmed by our data for both *E. coli* and *B. subtilis* (Figures 3 B and 3C).

Addition of Constant Size and Exponential Elongation Explain Correlations
The constant $\Delta$ model predicted that autocorrelations of $s_b$, $s_d$, and $t_d$ decayed by a factor of two in each generation and that the correlation coefficient between the generation time of the mother and its daughters was $-1/4$, which was also confirmed by the data (Figure S4). Intuitively, the negative correlation reflects the increased generation time of the daughter cells that were born smaller than $s_b$ due to stochastic, premature division of the mother cell [4]. Since all cells elongated exponentially with the elongation rate proportional to the cell length, cells born at $s_b < S_0$ would require more time to elongate by $\Delta$ for division than cells born at $s_b > S_0$ (Figure 1D, left, dashed line). Distributions of the Growth and Division Parameters Collapse when Rescaled by Their Respective Means
The constant $\Delta$ model in fact provides a quantitative explanation for the distributions of quantities involved in growth and size control. The six distributions of the relative septum position $s_{1/2}$, elongation rate $a$, division size $s_d$, newborn size $s_b$, generation time $t_d$, and size increment $\Delta$ are shown in Figure 4A. The coefficients of variation (CVs) of four distributions are related in the $\Delta$ model as

$$
\frac{\sigma_{s_b}}{\Delta} = \frac{1}{2^{1/4}} \frac{\sigma_s}{\sigma_t} = 2 \left( \frac{\sigma_{s_d}}{\sigma_{s_b}} \right) \geq 3 \left( \frac{\sigma_d}{\sigma_{s_d}} \right)^2 \quad \text{(Equation 1)}
$$

where $\sigma$ denotes the SD of the distribution (see theory section in Supplemental Information for details). This predicted hierarchy of variability was confirmed by our data for both *E. coli* and *B. subtilis* (Figure 4B). Note that the size at birth $S_0$ was slightly more variable than the size at division $S_d$ because of the small variability of the septum position $s_{1/2}$. The elongation rate $a$ was subject to its own physiological control and variability and showed negligible correlations with the distributions determined by $\Delta$ (Supplemental Information).

The constancy of $\Delta$ was finally supported by the scale invariance of the distributions shown in Figure 4A. In the constant $\Delta$ model, the average of the three size variables are related as $\langle \Delta \rangle = \langle S_0 \rangle/2$ and, if $r(\Delta)$ shows scale invariance, the three distributions $r(s_b)$, $r(s_d)$ and $r(t_d)$ also inherit the property of scale invariance of $r(\Delta)$ (theory section in Supplemental Information). In support of this theoretical prediction, all experimental $r(\Delta)$ and other size distributions collapsed onto each
ome data for the relative fraction of proteins; and P, containing the rest of the proteins. Using proteome data for the relative fraction of P-sector proteins in E. coli (Figure 4E, left; [11]) and the respective average cell volume (V) (Figure 1C, red line), we found that the total number of P-sector proteins per cell $N_\text{p} = \varphi_\text{p} \times (V)$ is relatively constant in all growth conditions for different E. coli strains (see Figure 4 and Supplemental Information). Thus, proteins in the P sectors behave as the initiators postulated in [22]. This leads to the prediction that the majority of proteins involved in metabolism (e.g., nutrient transporters and metabolic sensors [15]) and the cell cycle should belong to the P sector of the proteome (with their constant basal level to the Q sector). Note that the total proteome per cell increases exponentially with respect to the average growth rate; the growth law ([5]; Figure 1C) can thus be interpreted as a response of the average cell size (total proteome per cell) to nutrient conditions such that the average P-sector proteins per cell is approximately constant with respect to the nutrient-imposed growth rate. There is a clear experimental avenue for the future that will investigate how $\Delta$ will change when the proteome composition is perturbed by, e.g., transcription or translational inhibitors.

Extension to Other Organisms
The growing number of modern single-cell data sets provides a unique opportunity to determine the applicability of our findings to other bacteria as well as to eukaryotes. Analysis of bacteria, such as Caulobacter [25, 26], and single-celled eu- karyotes should illuminate the role played by programmed degradation of regulatory proteins in cell-size homeostasis. Fantes [27] considered structural models for fission yeast S. pombe and dismissed them based on existing data sets. While differences might indeed be expected between eukaryotes and bacteria, extensive modern single-cell data sets are now available in, e.g., budding yeast [28], and could be used to address the question [26]. It will also be of great interest to determine whether other non-rod-shaped organisms, particularly those that exhibit tip growth and/or nonuniform morphologies, including mycobacteria, hyphal fungi, and protists like Stentor, also add constant volume or maintain their size through other independent mechanisms. We finally remark that the size and the shape of cells play a major role in their physiology in multicellular organisms as well, namely during Xenopus embryogenesis [29].

Other (Figure 4A; [23, 24]). Hence, the variation of all the statistics with growth conditions is determined by the unique parameter $\Delta$.

Discussion
Proteome and Biological Origin of Constancy of Added Size
Since the proteome is a good proxy for cell size, the constant $\Delta$ is consistent with the "structural models" discussed by Fantes et al. [22]. Key features of the structural models include the following: (1) individual cells elongate exponentially, (2) initiators of cell cycle are produced at the same rate as the cell elongation rate, and (3) accumulation of the initiators to a threshold triggers the cell cycle [22]. Since the cellular volume and total number of proteins increase with the growth rate, the cellular fraction of protein initiators should reduce to maintain the constancy of the threshold. In a recent work by Scott et al. [11], the bacterial proteome is partitioned into three "sectors": R, containing ribosomal proteins; Q, containing housekeeping proteins; and P, containing the rest of the proteins. Using proteome data for the relative fraction $\varphi_\text{p}$ of the P-sector proteins in E. coli (Figure 4E, left; [11]) and the respective average cell volume (V) (Figure 1C, red line), we found that the total number of P-sector proteins per cell $N_\text{p} = \varphi_\text{p} \times (V)$ is relatively constant in all growth conditions for different E. coli strains (see Figure 4 and Supplemental Information). Thus, proteins in the P sectors behave as the initiators postulated in [22]. This leads to the prediction that the majority of proteins involved in metabolism (e.g., nutrient transporters and metabolic sensors [15]) and the cell cycle should belong to the P sector of the proteome (with their constant basal level to the Q sector). Note that the total proteome per cell increases exponentially with respect to the average growth rate; the growth law ([5]; Figure 1C) can thus be interpreted as a response of the average cell size (total proteome per cell) to nutrient conditions such that the average P-sector proteins per cell is approximately constant with respect to the nutrient-imposed growth rate. There is a clear experimental avenue for the future that will investigate how $\Delta$ will change when the proteome composition is perturbed by, e.g., transcription or translational inhibitors.

Extension to Other Organisms
The growing number of modern single-cell data sets provides a unique opportunity to determine the applicability of our findings to other bacteria as well as to eukaryotes. Analysis of
perspective on this issue and in the search for the underlying molecular mechanisms. A direction to be pursued in the future is the constancy of the added size $D$ and its relationship with the proteome \[11\]. That hints at an ensemble of molecular players and entails both exponential dependency of the average cell size on growth rate (the growth law) and constancy of $D$ at steady state. It will thus be important to interfere with protein synthesis and assess the resulting effects on the cell-size distributions.

Experimental Procedures

Strains
For physiological study, it is important to use a prototrophic strain. For \textit{E. coli}, we chose the strain K12 NCM3722, constructed, sequenced, and extensively tested by Sydney Kustu’s laboratory \[32\]. We used SJ202, a nonmotile derivative of NCM3722 (ΔmotA). For \textit{B. subtilis} experiments, we chose a strain in the 3610 background with Coml (Q12L) mutation to allow competence. We used a derivative with reduced motility and biofilm formation by deleting \textit{epsH} and a flagellin protein \textit{hag}, respectively.

Growth Media
\textit{E. coli} growth experiments were performed in seven different nutrient conditions. The average generation time in these conditions evenly spanned from 17.1 to 51.4 min at 37 °C. The growth medium is based on MOPS, developed by Fred Neidhardt \[33\], and is commercially available from Teknova (http://www.teknova.com). \textit{B. subtilis} growth experiments were performed in four different growth conditions with average doubling times between 16.9 and 38.9 min. The details of the growth media are listed in Tables S1 and S2. Prior to growth of the cells in the microfluidics device, all cultures were grown in a 37 °C water bath shaker, shaking at 240 rpm.

Sample Preparations
All experimental steps—from inoculation to imaging—were performed at 37 °C ± 0.1°C. To this end, all equipment was stationed in a 5’ × 7’ environmental chamber to eliminate any side effect of temperature fluctuations in the cell growth and physiology. Within the chamber, the temperature distribution was homogeneous, with forced air circulation within ±0.1 °C, and constantly monitored at multiple locations. See Supplemental Information for more information.

Microscopy
Image acquisition and analysis were performed with an inverted microscope (Nikon Ti-E) equipped with Perfect Focus (PFS 3), a 100× oil immersion objective lens (NA 1.45), and white LED transmission light (TLED, Sutter Instruments, 400–700 nm), and an Andor NEO sCMOS camera was used for phase-contrast imaging. The illumination condition was 50 ms exposure with illumination intensity set at 10% of the maximum TLED intensity. The frequency of the time-lapse imaging was chosen such that about 20 or
more images were taken per generation time. Imaging in phase contrast eliminated potential artifacts common in fluorescence imaging. Analysis of the large number of phase-contrast images required development of custom high-throughput image analysis software as described in Supplemental Information.

Model for the $\Delta$ Control

We denote by $s$ the cell size along the elongating axis of the rod and by $s_0$ and $s_B$ the sizes of cells at birth and division. We assume the width of the cell is roughly constant. If $s(t)$ is the size of a cell at the current time $t$, its added size is denoted $\Delta(t) = s(t) - s_0$. The $\Delta$ model posits that the mechanism of control involves the single variable, $\Delta = s(t) - s_0$, the size added between birth and division. The density of cells $n(s, \Delta, t)$ having size $s$ and added size $\Delta$, with $g(s)$ as $ds/dt$, obeys the continuity equations

$$\frac{\partial n(s, \Delta, t)}{\partial t} + \frac{\partial}{\partial s} [g(s)n(s, \Delta, t)] + \int_0^\infty \gamma(\Delta) j(s, t) dx = \frac{\partial n(s, \Delta, t)}{\partial \Delta}$$

(2)

$$g(s)n(s, 0, t) = 4g(2s) \int_0^\infty \gamma(x)n(2s, x, t) dx$$

(3)

The left-hand side in Equation 2 is the total time derivative, and the two drift terms are due to the elongation of the cells, i.e., $ds/dt = g(s)$ and $d\Delta(t)/dt = g(s)$. The right-hand side accounts for the division of cells. The Poissonian splitting rate function $\gamma(\Delta)$ is related to the distribution $\rho_{ss}(\Delta)$ for the size added at division of individual cells as $n(\Delta) = \gamma(\Delta) \exp(-\gamma(x) dx)$. Indeed, the exponential term is the probability that the cell will not divide up to $\Delta$ and the $\Delta dx$ is the probability of division in the range $(\Delta, \Delta + dx)$. Simple algebra leads then to

$$\gamma(\Delta) = \frac{\rho_{ss}(\Delta)}{1 - \int_0^\infty \rho_{ss}(\delta) dx}$$

(4)

The conversion of the rate of division to unit time involves the Jacobian $d\Delta(t)/dt = g(s)$ that appears in the right-hand side of Equation 2. Finally, Equation 3 is the boundary condition that accounts for cells having all $\Delta = 0$ at birth, irrespective of their size $2s$ at division. Equation 2 goes back at least to [34, 35], and the formalism was then expanded and utilized for the sized, the timer, and their combinations in a series of papers and books (see, e.g., [8, 24, 36–39]). We took the pragmatic approach of extracting the functions $g$ and $\gamma$ from the distribution of the sizes at division and of the elongation rates and using them to simulate the cell-size control process at the level of individual cells. We then compared statistical observables alternative to those used for the calibration of the model. As detailed in the Supplemental Information, this procedure allowed us to rule out timer and size models and to establish the consistency of the $\Delta$ model.

Supplemental Information

Supplemental Information includes Supplemental Experimental Procedures, four figures, three tables, and one movie and can be found with this article online at http://dx.doi.org/10.1016/j.cub.2014.12.009.

Author Contributions


Acknowledgments

This work was supported by the Paul G. Allen Foundation, the Pew Charitable Trusts, and the National Science Foundation CAREER Award (to S.J.) and by the NIH grant GM 64671 (to P.A.L.).

Received: October 24, 2014
Revised: November 23, 2014
Accepted: December 2, 2014
Published: December 24, 2014

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Update

Current Biology
Volume 27, Issue 9, 8 May 2017, Page 1392

DOI: https://doi.org/10.1016/j.cub.2017.04.028
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Sattar Taheri-Araghi, Serena Bradde, John T. Sauls, Norbert S. Hill, Petra Anne Levin, Johan Paulsson, Massimo Vergassola,* and Suckjoon Jun*

*Correspondence: massimo@physics.ucsd.edu (M.V.), suckjoon.jun@gmail.com (S.J.)
http://dx.doi.org/10.1016/j.cub.2017.04.028

(Current Biology 25, 385–391; February 2, 2015)
We have become aware of an error in Equation 1 and Figure 4B of this article. The corrected Equation 1 should read as follows, to match Equation 25 in the Supplemental Information, which presents a full derivation of the correct Equation 1 without the error:

$$\frac{\sigma_{\Delta}}{\langle \Delta \rangle}^2 = 3 \cdot \ln 2 \cdot \left( \frac{\sigma_d}{\langle S_b \rangle} \right)^2 = 3 \cdot \left( \frac{\sigma_{d}}{\langle S_b \rangle} \right)^2 = 3 \cdot \left( \frac{\sigma_d}{\langle S_b \rangle} \right)^2 .$$

(Equation 1)

Figure 4B with a corrected annotation is shown below. The data in Figure 4B remain unchanged.

Please note that these errors do not impact the results of our study. The authors apologize for any confusion.

![Figure 4B. Origin and Quantitative Consequences of Constancy of Added Size Δ](image-url)